Optimal consumption choice under a CES consumption bundle

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Household consumption C_t is a composite index of domestic and imported consumption, $C_{h,t}$ and $C_{f,t}$,

$$C_t = \left[(1-\phi)^{\frac{1}{\zeta}} C_{h,t}^{\frac{\zeta-1}{\zeta}} + \phi^{\frac{1}{\zeta}} C_{f,t}^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{1}$$

where ϕ is the import share of domestic consumption and ζ the elasticity of substitution between domestic goods and imports. Domestic and imported consumption $C_{h,t}$ and $C_{f,t}$ are sold at prices $P_{h,t}$ and $P_{f,t}$, respectively, while the overall consumer price index is P_t:

$$P_t C_t = P_{h,t} C_{h,t} + P_{f,t} C_{f,t}$$

To derive optimal levels of $C_{h,t}$ and $C_{f,t}$, households solve

$$\max_{C_{h,t}, C_{f,t}} P_t C_t - P_{h,t} C_{h,t} - P_{f,t} C_{f,t} \text{ s.t. (1)}$$

This gives the two first-order conditions

$$C_{h,t} = (1-\phi)\left(\frac{P_{h,t}}{P_t}\right)^{-\zeta} C_t$$
$$C_{f,t} = \phi\left(\frac{P_{f,t}}{P_t}\right)^{-\zeta} C_t.$$

Moreover, substituting these conditions back into (1), we obtain an expression for the consumer price P_t in terms of the price of domestic and foreign goods:

$$P_t = \left[(1-\phi) P_{h,t}^{1-\zeta} + \phi P_{f,t}^{1-\zeta}\right]^{\frac{1}{1-\zeta}},$$
$$\text{or } 1 = (1-\phi)\left(\frac{\bar{P}_{h,t}}{P_t}\right)^{1-\zeta} + \phi\left(\frac{\bar{P}_{f,t}}{P_t}\right)^{1-\zeta}.$$